forall F. unitresolution(f,{}) -> forall assignment.  F[assignment] = false (soundness) - something like this

*Special case – separate case*

Assumptions on whether the formula is valid etc?

**Goal:** Unit Resolution (f, c) -> Logical Entailment (f, c)

*If c can be derived from f with Unit Resolution, then c must be logically entailed from f.*

Assume m is a model of f, and c is a member of f, if c is derived from f with unit resolution then c is we have Logical Entailment

**Proof Description:**

Unfolding Definitions: We expand the definition of entailment.

Assumptions: Let "unitres f c1" be true, indicating the existence of a unit resolution derivation from formula "f" to clause "c1". This implies that clause "c1" can be derived from "f" using unit resolution.

Induction on unit res: 2 cases

Subsumption Case:

Unfolding Definitions: We expand the definitions of "models\_formula" and "models\_clause".

Assumptions:

We assume the existence of a model "H\_model\_clause" that satisfies the "clause".

"h\_model\_clause\_prop" represents the property that the model "H\_model\_clause" satisfies clause "c".

"h\_model\_prop" is a proposition derived from "h\_model\_clause\_prop" and is used to establish the entailment of "c".

"h\_model\_clause\_prop2" is another property related to the model "H\_model\_clause", which may be relevant for further reasoning within the subsumption case.

Application of Hypothesis: We specialize "h\_model\_prop" with clause "c".

Specialization: We specialize "h\_model\_prop" with arguments "h\_model\_clause\_prop", "H", and "h\_model\_clause\_prop" (named as "h\_model\_prop\_specialized").

Assumptions: We introduce variables "l0" and "l0prop" to represent certain properties.

Destructing: We use the destruct tactic to examine the properties of the clause "c" and "c2" under the given model assumptions.

Application of Proposition: We apply the derived proposition.

Existential Introduction: We introduce another existential variable "x".

Application of Proposition: We apply the derived proposition.

Conclusion: Thus, "f" entails "c".

Resolution Case: (To be completed)

Application of Lemma:

We apply the lemma: "f entails c" and "f entails neg l" imply "f entails c \ l".

This lemma helps establish the entailment of "f" to the result clause "remove\_literal\_from\_clause l c".

Prove the Goal:

Given the induction hypotheses IHunitres\_c1\_1 and IHunitres\_c1\_2, which state that for any model "m" satisfying certain properties, "f" entails "c" and "f" entails "c2", respectively:

We need to show that for any such model "m" satisfying the properties:

If "f" entails "c" and "f" entails "c2", then "f" entails "remove\_literal\_from\_clause l c" (the goal clause).

To achieve this, we use the result of the lemma and the given induction hypotheses:

We know that "f" entails "c" (from IHunitres\_c1\_1) and "f" entails "c2" (from IHunitres\_c1\_2).

By applying the lemma, we conclude that "f" entails "c \ l".

Thus, "f" entails "remove\_literal\_from\_clause l c", which completes the proof of the goal.

This completes the proof of the goal.

Conclusion: Therefore, if "unitres f c" holds, then "f" entails "c".

**Proof Outline:**

Eliminate Inductive definitions to have the two cases:

Subsumption:

Resolution:

Subsumption: Show that if the empty clause is derived from a single formula via subsumption, then the formula itself must be unsatisfiable. This is trivial because the empty clause directly indicates unsatisfiability.

Resolution: Assume that the empty clause can be derived from two unit resolutions, R1​ and R2​, and by the inductive hypothesis, both R1 and R2 lead to unsatisfiability.

* Prove that the resolution operation preserves unsatisfiability, if both R1 and R2 lead to unsatisfiability, then the result of resolution also leads to unsatisfiability.

**Conclusion:**

By induction, since the base case holds and the inductive step preserves the property, we conclude that for any derivation of the empty clause using the unitres constructors, there is no assignment to the variables of the formula that makes it true.

Proof:

Let *f* represent a set of clauses in propositional logic, and let *c* represent a clause. We aim to prove that if *c* can be derived from *f* using Unit Resolution, denoted as (*f*,*c*), and if *c*2​ is a unit clause in *f*, then *c* is logically entailed by *f*.

Assume *c* can be derived from *f* using Unit Resolution, i.e., (*f*,*c*). This means there exist clauses *c*1​ and *c*2​ in *f* such that *c* can be obtained by resolving *c*1​ and *c*2​.

Now, suppose *f* is logically entailed, i.e., there exists a model *M* such that every clause in *f* evaluates to true under *M*.

Since *c*2​ is a unit clause in *f*, its literal must be true under *M*. Without loss of generality, let's assume *c*2​ is of the form {*p*} where *p* is a literal.

Because *c*1​ and *c*2​ are clauses in *f*, they evaluate to true under *M*. By the definition of Unit Resolution, resolving *c*1​ and *c*2​ to obtain *c* maintains logical consistency. Thus, *c* also evaluates to true under *M*, which means *c* is logically entailed by *f*.

Therefore, we have shown that if *c* can be derived from *f* using Unit Resolution and *c*2​ is a unit clause, then *c* is logically entailed by *f*. This completes the proof.

**Subsumption Case**:

* + Show that if the empty clause is derived from a single formula via subsumption, then the formula itself must be unsatisfiable.
  + This is indeed trivial because the empty clause directly indicates unsatisfiability.

1. **Resolution Case**:
   * Assume that the empty clause can be derived from two unit resolutions, *R*1​ and *R*2​.
   * By the inductive hypothesis, both *R*1​ and *R*2​ lead to unsatisfiability.
2. **Preservation of Unsatisfiability under Resolution**:
   * Prove that the resolution operation preserves unsatisfiability.
   * If both *R*1​ and *R*2​ lead to unsatisfiability, then the result of resolution also leads to unsatisfiability.

**Detailed Proof:**

**Base Case:**

Subsumption: Let ϕ be a formula. If the empty clause is derived from ϕ by subsumption, it means ϕ itself is unsatisfiable. This is evident because the empty clause represents unsatisfiability, so no assignment to the variables in ϕ can make it true.

**Inductive Step:**

Resolution: Assume that the empty clause is derived from two unit resolutions R1 and R2. By the inductive hypothesis, both R1 and R2 lead to unsatisfiability. We want to show that the result of resolution on R1 and R2 also leads to unsatisfiability.

Let's denote the formulas involved in R1 and R2 as ϕ1 and ϕ2 respectively.

Through resolution, the result would contain clauses derived from ϕ1 and ϕ2, along with resolution steps. Since both R1 and R2 lead to unsatisfiability, every clause in their derivation is unsatisfiable.

The resolution step only combines clauses in a way that preserves unsatisfiability. This is because the resolution operation is sound, meaning that if its inputs are unsatisfiable, its output will also be unsatisfiable.

**Conclusion:**

By induction, since the base case holds and the inductive step preserves the property, we conclude that for any derivation of the empty clause using the unitres constructors, there is no assignment to the variables of the formula that makes it true.